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LETTER TO THE EDITOR

Oscillatory transient velocity of miniband transport in one-dimensional superlattices

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Abstract. The transient drift velocity response of miniband transport to a step electric field is investigated for one-dimensional semiconductor superlattices using the balance equation approach. Bloch-type oscillatory behaviour appears at a field having strength larger than about eight times the critical field in the steady-state conduction. With increasing field strength, an increasing number of oscillations survive before a steady value is finally approached. In one-dimensional GaAs-based systems the field strength required to observe an oscillatory miniband transient response is of the order of $10^3\,\mathrm{V\,cm^{-1}}$, well within the range where the semiclassical description of transport applies.

Negative differential velocity (NDV) in superlattice vertical transport, predicted years ago by Esaki and Tsu [1], has been demonstrated experimentally [2, 3]. Detailed theoretical investigations of steady-state linear and non-linear miniband conduction have also been carried out for three-, two- and one-dimensional superlattices [4–7] using the balance equation approach [8, 9]. In an ideal superlattice, a uniform electric field drives a scattering-free (ballistic) electron to perform Bloch oscillation. In real systems, however, elastic and phonon scattering, which are unavoidable under the influence of a high electric field even at zero lattice temperature, usually suppress the oscillatory transient velocity even when the applied electric field strength is suddenly brought up to the NDV region of steady-state conduction. In the case of a three-dimensional superlattice, the preliminary calculation from the balance-equation theory found a pronounced velocity overshoot, but no Bloch-type oscillation showing up in the transient response to a suddenly turned-on electric field as high as $10\,\mathrm{kV\,cm^{-1}}$ [6, 10]. On the other hand, calculations based on the one-dimensional (1D) Boltzmann equation with two (elastic and inelastic) constant scattering times [11], indicated the possibility of oscillatory transient behaviour for specific ranges of parameters. Unfortunately, the phenomenological elastic and inelastic scattering times introduced in the constant-relaxation-time approximation, are not realistic and are impractical to estimate from the electron–phonon coupling information of the materials. The purpose of this letter is to investigate the transient response of a one-dimensional superlattice to step electric fields of differing strengths, using the balance equation theory. Since realistic electron–impurity and electron–phonon scatterings

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are taken into full account in the balance equation theory, transport properties can be evaluated from the known scattering information of the material. It is thus possible to judge under what conditions (such as lattice temperature, low-temperature linear mobility and electric field strength) the miniband transient response would be oscillatory.

We consider the same model superlattice system as discussed in [7]: electrons can move along the z-direction through the (lowest) miniband formed by the periodic potential wells and barriers of finite height, and are confined in the transverse (x–y) plane to a small cylindrical region of diameter d_r. To facilitate comparison with 1D Boltzmann-type theory, only the lowest transverse subband is taken into account in the present model. The electron energy dispersion is given, under the tight-binding approximation, by

$$\varepsilon(k_z) = (\Delta/2)(1 - \cos k_z d) \tag{1}$$

Δ being the miniband width.

When a time-dependent uniform electric field E is applied parallel to the superlattice axis (z-direction), the carriers are accelerated by the field and scattered by impurities and by phonons, resulting in an overall drift motion and possible heating and cooling of the carrier system. Such a transport state is described in the balance equation theory by the centre-of-mass (CM) momentum $P_d \equiv N p_d$ (N is the total number of carriers) and the relative electron temperature T_e, and they are determined by the effective force and energy balance equations:

$$\mathrm{d}v_d/\mathrm{d}t = eE/m_z^* + A_i + A_p \tag{2}$$

$$\mathrm{d}h_e/\mathrm{d}t = eEv_d - W. \tag{3}$$

Here

$$v_d = \frac{2}{N}\sum_{k_z} \frac{\mathrm{d}\varepsilon(k_z)}{\mathrm{d}k_z} f(\bar{\varepsilon}(k_z), T_e) \tag{4}$$

is the CM velocity, or the average drift velocity of the carriers,

$$\frac{1}{m_z^*} = \mathcal{K}_{zz} = \frac{2}{N}\sum_{k_z} \frac{\mathrm{d}^2\varepsilon(k_z)}{\mathrm{d}k_z^2} f(\bar{\varepsilon}(k_z), T_e) \tag{5}$$

is the inverse effective mass of the CM, and

$$h_e = \frac{2}{N}\sum_{k_z} \varepsilon(k_z) f(\bar{\varepsilon}(k_z), T_e) \tag{6}$$

is the average electron energy in the miniband. In these equations

$$f(\varepsilon, T_e) \equiv \{\exp[(\varepsilon - \mu)/T_e] + 1\}^{-1} \tag{7}$$

is the Fermi distribution function at the electron temperature T_e, μ is the chemical potential determined by the condition that the total number of electrons equals N:

$$N = 2\sum_{k_z} f(\bar{\varepsilon}(k_z), T_e) \tag{8}$$

$$\bar{\varepsilon}(k_z) \equiv \varepsilon(k_z - p_d) \tag{9}$$

is the relative electron energy. The expressions for the impurity- and phonon-induced frictional accelerations, A_i and A_p, and the energy-transfer rate from the electron

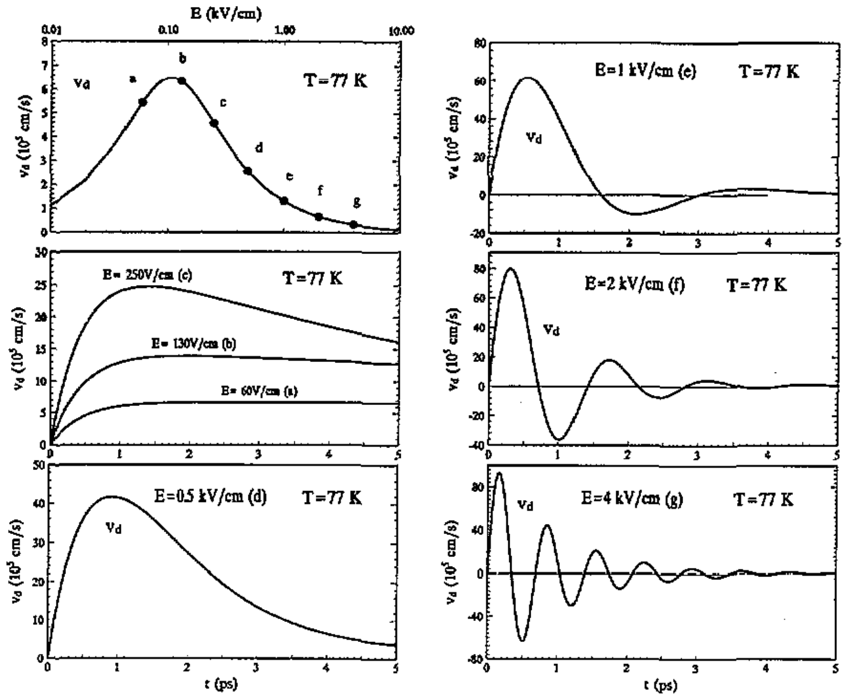

Figure 1. Steady-state drift velocity as a function of applied electric field (upper left corner), and the transient drift velocity response to step electric fields turned on at time $t = 0$ with different strengths (a)–(f) for a one-dimensional superlattice of period $d = 15\,\text{nm}$, miniband width $\Delta = 220\,\text{K}$, transverse diameter $d_r = 10\,\text{nm}$, and carrier line density $N_1 d = 0.314$. Calculations are carried out at lattice temperature $T = 77\,\text{K}$.

system to the phonon system, W, were given in [7], together with the form factors due to longitudinal and transverse wavefunctions.

The energy dispersion form (1) enables us to write ($v_m = \Delta d/2$, $1/M^* = \Delta d^2/2$)

$$v_d = v_m \alpha_T^{(1)} \sin z_d \tag{10}$$

$$1/m^* = (1/M^*)\alpha_T^{(1)} \cos z_d \tag{11}$$

$$h_e = \Delta/2 - (\Delta/2)\alpha_T^{(1)} \cos z_d \tag{12}$$

with $\alpha_T^{(1)}$ defined by

$$\alpha_T^{(1)} = \frac{2}{N} \sum_{k_z} \cos(k_z d) f(\varepsilon(k_z), T_e) \tag{13}$$

which is a function of T_e, but independent of the parameter $z_d \equiv p_d d$. With the help of these expressions we can write the balance equations (2) and (3) for a 1D-superlattice in the following simple form

$$dz_d/dt = eEd + \{[2(A_i + A_p)/\Delta d] \cos z_d - (2W\Delta) \sin z_d\}/\alpha_T^{(1)} \tag{14}$$

$$dT_e/dt = \{[2(A_i + A_p)/\Delta d] \sin z_d + (2W/\Delta) \cos z_d\}(d\alpha_T^{(1)}/dT_e)^{-1}. \tag{15}$$

This coupled set of differential equations is easily solved numerically once the frictional acceleration A_i and A_p, and the energy-transfer rate W are calculated as functions of z_d and T_e [7].

As an example we have carried out numerical calculation at lattice temperature $T = 77\,\mathrm{K}$ for a one-dimensional GaAs-based superlattice of period $d = 15\,\mathrm{nm}$, miniband width $\Delta = 220\,\mathrm{K}$, transverse diameter $d_r = 10\,\mathrm{nm}$, and low-temperature (4.2 K) linear mobility $\mu_0 = 1.0\,\mathrm{m^2\,V s^{-1}}$. We assume a carrier sheet density of $N_s = 4.0 \times 10^{15}\,\mathrm{m^{-2}}$ (per period) in the transverse plane, corresponding to a line density $N_1 d = 0.314$. For this narrow miniband width the polar optic phonon scattering is prohibited by the requirement of energy conservation. This greatly reduces the critical field E_c at which the steady-state drift velocity peaks, and enables the oscillatory transient response to appear at a field when the Stark quantization energy eEd is less than the miniband width Δ. Both longitudinal and transverse acoustic phonons (through the deformation potential and piezoelectric couplings with electrons) are included in the calculation. The calculated steady-state velocity–field curve is shown in the upper left corner of figure 1. The transient velocity responses to step electric fields turned on at time $t = 0$ having strengths (a) $E = 60\,\mathrm{V\,cm^{-1}}$, (b) $E = 130\,\mathrm{V\,cm^{-1}}$, (c) $E = 250\,\mathrm{V\,cm^{-1}}$, (d) $E = 0.5\,\mathrm{kV\,cm^{-1}}$, (e) $E = 1\,\mathrm{kV\,cm^{-1}}$, (f) $E = 2\,\mathrm{kV\,cm^{-1}}$ and (g) $E = 4\,\mathrm{kV\,cm^{-1}}$, are shown respectively on the other parts of the figure. The steady-state critical field E_c is about $105\,\mathrm{V\,cm^{-1}}$ for this system. In the case of (a) $E < E_c$, there is no velocity overshoot in the transient response. Once $E > E_c$, (cases (b), (c) and (d)), the velocity overshoot shows up, and becomes more pronounced at larger E. The oscillatory transient velocity begins to appear at a field strength about eight times the critical field E_c. At $E = 1\,\mathrm{kV\,cm^{-1}}$ clear oscillations of the drift velocity are already seen, with the oscillatory period equal to 3.0 ps, a little larger than the free Bloch oscillation period $T_B = 2\pi\hbar/(eEd) = 2.76\,\mathrm{ps}$ at this field. When the field strength further increases, the transient velocity oscillation gets increasingly stronger ((f) and (g)). For the cases of $E = 2\,\mathrm{kV\,cm^{-1}}$ and $E = 4\,\mathrm{kV\,cm^{-1}}$, the oscillation periods are 1.41 ps and 0.695 ps respectively, and the corresponding free Bloch periods are $T_B = 1.38\,\mathrm{ps}$ and 0.69 ps. Even at this largest field $E = 4\,\mathrm{kV\,cm^{-1}}$, the Stark quantization energy $eEd/k_B = 69.6\,\mathrm{K}$ is still much smaller than the miniband width $\Delta/k_B = 220\,\mathrm{K}$, indicating that the system is well within the range where a semiclassical description of transport applies. It seems that one-dimensional superlattices are good candidates for observing oscillatory transient drift velocity in miniband transport.

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